

Collaboration Mechanism Design under Data Uncertainty in Multicommodity Flow Networks

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Introduction A network is called a collaborative one if the network users share edge resources. We study a collaborative multicommodity flow (MCF) network model with multiple source-sink pairs. Each shipped commodity creates a revenue per unit. Both the edge capacity and commodity demand are owned by individual users in this network system. We also allow multiple owners of capacity on a single edge. Individuals are assumed to be selfish in the sense that they design routing solutions for their own commodities in order to maximize their profits. Practical examples of such MCF networks include transportation networks, Internet and telecommunication networks.

In general, routing schemes in MCF networks which maximize the social welfare, *i.e.*, the total shipping revenue, are desired. However, since in real situations the operation of such networks commonly depends on routing decisions made by selfish individuals, we often need collaboration mechanisms to subtly regulate selfishness so that everyone is willing to cooperate towards the best system outcome.

It is shown in [1] that a capacity exchange price mechanism can be used to manage such MCF networks. Specifically, the mechanism allows individuals to buy/sell edge capacity over the counter at exchange prices imposed by a central authority. Such prices alter the final profit of each network user and thus modify his behaviour. It is shown that prices can be computed so that individuals are directed to follow the socially optimal routing if forced to stay in the grand coalition, for instances, by binding agreements. Moreover, under certain conditions, cooperation stability can also be ensured under specific prices in the sense that no one profits less than what can be achieved on his own and thus leaves the grand coalition [1] [2].

However, the above results only exist for static MCF networks with fixed edge capacity, commodity demand and unit shipping revenue. In real applications some fluctuations of such network parameters are to be expected over time. Since it is impossible to alter the mechanism every time there is such a change, we need one mechanism that is effective in promoting the social welfare not only in a specific scenario with a fixed network, but also under a series of realizations of network parameters.

Project We target a stochastic network with variable network parameters with given variation ranges but no prior knowledge of their probability distribution. We try to design mechanisms so that its induced routing scheme synthesized by selfish individual decisions of their own commodities generates fair total revenue under any realization of network parameters. However, our initial study suggests that the capacity exchange mechanisms with fixed prices may give rise to an infeasible routing in the network under variable edge capacity and commodity demand [3]. In the light of this observation, our first task is to design *robust* mechanisms under which the resulting routing stays *feasible* under any possible scenario. Moreover, notice that the induced routings under robust mechanisms may become *sub-optimal* in terms of the total shipping revenue generated. This sacrifice of social optimality can be defined as the price of robustness (*PoR*) under the robust mechanisms and characterized by the ratio between the total revenue under the socially optimal routing and the induced one. Since we assume that no probability distribution of the parameters is known, we take up the worst-case analysis to the *PoR* ratio and try to give upper bounds to its value over a range of network scenarios.

Our first study to this problem is made under the assumption that individuals are forced to stay in the grand coalition but are free to make capacity management and commodity routing decisions of their own network resources. It indicates that given certain assumptions on individual ownership of edge capacity in the network, the mechanism based on fixed capacity exchange prices can be made robust under resource level variations by redesigning the network structure or incorporating capacity

control mechanisms [3]. The *PoR* bound under either of the approaches is shown to depend on the variation ranges and certain network properties, and thus can be small under specific conditions. Based on these previous results, I am interested in further studying the following topics.

1. We have shown that redesigning the network structure helps to reinforce the robustness of the capacity exchange price mechanisms if the resulting network satisfies certain conditions. We also gave simple examples in [3] to demonstrate how such network design can be done. However we still need to formulate an efficient algorithm to perform the task on general networks, which is one of our next-step goals.
2. Since the mechanisms given by our initial study are only shown to be robust under specific assumptions on individual ownership of edge capacity, we need to study on how these mechanisms can be modified to be robust based on *general* individual ownerships. Taken into account the price of robustness, such modifications should also be designed so that the upper bound of the resulting *PoR* ratios is not too high.
3. Besides to induce individuals towards the socially optimal routing, another issue in promoting the social welfare is how to guarantee the stability of such cooperation if individuals are free to join/leave the grand coalition. While in static networks the stability of the grand coalition can be ensured by the capacity exchange mechanism under specific prices or network conditions, this is an open question in stochastic networks. Unfortunately under either of the two mechanisms introduced in [3], we can find easy examples in which network users tend to turn away from the coalition, as individual optimality is much jeopardized to build up group feasibility in the network. Hence we may approach the above issue by characterizing the loss of the coalition stability under these two mechanisms in general stochastic networks, which may serve as guidelines for the better design of robust mechanisms.

In summary, our project aims at formulating effective mechanism designs to promote cooperation among selfish network users towards the best system outcome in stochastic networks. Meanwhile, we also try to understand the influence of various factors, such as certain network properties, on the design of robust mechanisms as well as on the loss of the social optimality under the resulting mechanisms.

References

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